Introduction to Trigonometry

1.

If $\sin\theta = \frac{1}{3}$, then $\sec\theta$ is equal to :

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{3}{2\sqrt{2}}$ (c) 3 (d) $\frac{1}{\sqrt{3}}$ **(2024)**

Answer. (b) $\frac{3}{2\sqrt{2}}$

2. For what value of θ, $\sin^2\theta$+ sine+$\cos^2\theta$ is equal to 2 ? (2024)

(a) 45°
(b) 0°
(c) 90°
(d) 30°

Answer. (c) 90°

3. In a ΔABC, <A = 90°. If tan C =√3, then find the value of sin B+ cos C-$\cos^2$ B. (2024)

Answer.

$\tan C = \sqrt{3} \Rightarrow \angle C = 60° \Rightarrow \angle B = 30°$

$\therefore \sin B + \cos C - \cos^2 B = \frac{1}{2} + \frac{1}{2} - \frac{3}{4} = \frac{1}{4}$

4.

Prove that : $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \operatorname{cosec} A$ **(2024)**

Answer.

$$\text{LHS} = \frac{\sec A - 1 + \sec A + 1}{\sqrt{\sec^2 A - 1}}$$

$$= \frac{2 \sec A}{\tan A}$$

$$= 2 \operatorname{cosec} A = \text{RHS}$$

5. If sectan = m, then the value of secθ + tanθ is: (2024)

(a) $1 - \frac{1}{m}$ (b) $m^2 - 1$ (c) $\frac{1}{m}$ (d) $-m$

Answer.

(c) $\frac{1}{m}$

6.

If $\cos(\alpha + \beta) = 0$, then value of $\cos\left(\frac{\alpha + \beta}{2}\right)$ is equal to :

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 0 (d) $\sqrt{2}$

(2024)

Answer.

(a) $\frac{1}{\sqrt{2}}$

7. (A) Evaluate: $2\sqrt{2} \cos 45° \sin 30° + 2\sqrt{3} \cos 30°$ (2024)

Answer.

$2\sqrt{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{2} + 2\sqrt{3} \times \frac{\sqrt{3}}{2}$

$= 4$

OR

$\sin(A + B) = \sin A \cos B + \cos A \sin B$ (2024)

Answer.

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = 1$

$\therefore$ LHS = RHS

8.

Prove that : $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$ **(2024)**

Answer.

$$\text{LHS} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{(\sin\theta - \cos\theta)}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{(\cos\theta - \sin\theta)}{\cos\theta}}$$

$$= \frac{1}{(\sin\theta - \cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$= \frac{1}{(\sin\theta - \cos\theta)} \times \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} + 1$$

$$= 1 + \sec\theta \operatorname{cosec}\theta = \text{RHS}$$

Introduction to Trigonometry

Previous Years' CBSE Board Questions

8.2 Trigonometric Ratios

MCQ

1.

If 2 tan A = 3, then the value of $\dfrac{4\sin A+3\cos A}{4\sin A-3\cos A}$ is

(a) $\dfrac{7}{\sqrt{13}}$ (b) $\dfrac{1}{\sqrt{13}}$

(c) 3 (d) does not exit *(2023)*

2.

Given that $\cos\ \theta=\dfrac{\sqrt{3}}{2}$, then the value of $\dfrac{\text{cosec}^2\theta-\sec^2\theta}{\text{cosec}^2\theta+\sec^2\theta}$ is

(a) −1 (b) 1 (c) $\dfrac{1}{2}$ (d) $-\dfrac{1}{2}$

(Term I, 2021-22)

3.

$\dfrac{1}{\text{cosec}\theta(1-\cot\theta)}+\dfrac{1}{\sec\theta(1-\tan\theta)}$ is equal to

(a) 0 (b) 1

(c) $\sin\theta+\cos\theta$ (d) $\sin\theta-\cos\theta$

(Term I, 2021-22)

4. If sin 0 = cos 0, then the value of tan20+ $\cot^2$ 0 is
(a) 2
(b) 4
(c) 1
(d) 10/3 (2020C)

5.

If $\tan\theta + \cot\theta = \frac{4\sqrt{3}}{3}$, then find the value of $\tan^2\theta + \cot^2\theta$. *(2021C)*

SA I (2 marks)

6. Given 15 cot A = 8, then find the values of sin A and sec A. (2020C)

7.

If $3\cot A = 4$, prove that $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$.
(Board Term I, 2015)

SA II (3 marks)

8.

Given $\sin A = \frac{3}{5}$, find the other trigonometric ratios of the angle A. *(Board Term I, 2016)*

9.

If $3\tan A = 4$ check whether

$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.
(Board Term I, 2017)

8.3 Trigonometric Ratios of Some Specific Angles

MCQ

10.

$\left[\frac{5}{8}\sec^2 60° - \tan^2 60° + \cos^2 45°\right]$ is equal to

(a) $\frac{-5}{3}$ (b) $\frac{-1}{2}$ (c) 0 (d) $\frac{-1}{4}$
(2023)

11.

Given that $\sin\alpha = \frac{\sqrt{3}}{2}$ and $\tan\beta = \frac{1}{\sqrt{3}}$, then the value of cos (α – β) is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) 0 (d) $\frac{1}{\sqrt{2}}$

(Term I, 2021-22) Ap

12. The value of 0 for which 2 sin 20 = 1, is
(a) 15°
(b) 30°
(c) 45°
(d) 60° (Term I, 2021-22)

VSA (1 mark)

13. Evaluate:
2 sec 30° x tan 60° (2020)

14. Write the value of $\sin^2 30° + \cos^2 60°$. (2020)

15.
Evaluate :

$\frac{2\tan 45° \times \cos 60°}{\sin 30°}$ *(2020)*

16. If sinx + cosy = 1; x = 30° and y is an acute angle, find the value of y. (A/ 2019)

17.

If $\sin\alpha = \frac{1}{2}$, then find the value of $3\sin\alpha - 4\sin^3\alpha$.

(Board Term I, 2017) Ap

SAI (2 marks)

18. Evaluate 2sec20 + 3cosec20 - 2sinecose if 0 = 45° (2023)

19. If sine cose = 0, then find the value of $\sin^1 0 + \cos^1 0$. (2023)

20.

Evaluate : $\frac{5}{\cot^2 30°} + \frac{1}{\sin^2 60°} - \cot^2 45° + 2\sin^2 90°$

(2023)

21. If 0 is an acute angle and sine = cose, find the value of $\tan^2 0 + \cot^2 0 - 2$. (2023)

22. Take A = 60° and B = 30°. Write the values of
cosA + cosB and cos(A + B).
Is cos(A + B) = cosA + cosB? (Board Term 1, 2017)

23. Find cosec30° and cos60° geometrically. (Board Term 1, 2017)

24.

$\sin(A+B) = 1$ & $\sin(A-B) = \frac{1}{2}$,
$0 \le A + B = 90°$ & $A > B$, then find A & B.

(Board Term I, 2017)

LA (4/5/6 marks)

25. If 0=30°, verify the following:
(i) cos30 = $4\cos^3 0$ - 3cose
(ii) sin30 = 3sine - $4\sin^3 0$ (Board Term 1, 2017)

26. Find trigonometric ratios of 30° & 45° in all values of T.R. (Board Term 1, 2017)

27. If sin(A+B) = sinA.cosB + cosA.sinB and cos(A - B) = cosA.cosB + sinA.sinB
Find the value of (i) sin 75° (ii) cos 15° (Board Term 1, 2016)

8.4 Trigonometric Identities

MCQ

28. $(\sec^2 0 - 1)(\text{cosec}^2 0 - 1)$ is equal to
(a) -1
(b) 1
(c) 0
(d) 2 (2023)

29. Which of the following is true for all values of $\theta(0° \leq \theta \leq 90°)$?

(a) $\cos^2\theta - \sin^2\theta = 1$ (b) $\text{cosec}^2\theta - \sec^2\theta = 1$

(c) $\sec^2\theta - \tan^2\theta = 1$ (d) $\cot^2\theta - \tan^2\theta = 1$

(2023)

30.

Given that $\sin\theta = \frac{p}{q}$, $\tan\theta$ is equal to

(a) $\frac{p}{\sqrt{p^2 - q^2}}$ (b) $\frac{q}{\sqrt{p^2 - q^2}}$

(c) $\frac{p}{\sqrt{q^2 - p^2}}$ (d) $\frac{q}{\sqrt{q^2 - p^2}}$

(Term I, 2021-22)

31.

The simplest form of $\sqrt{(1 - \cos^2\theta)(1 + \tan^2\theta)}$ is

(a) $\cos\theta$ (b) $\sin\theta$ (c) $\cot\theta$ (d) $\tan\theta$

(Term I, 2021-22)

32. If $\sin^2 0 + \sin 0 = 1$, then the value of $\cos^2 0 + \cos^4 0$ is

(a) -1

(b) 1

(c) 0

(d) 2 (Term I, 2021-22)

33. The distance between the points (acose + bsine, 0) and (0, asino - bcose), is

(a) $a^2 + b^2$ (b) $a^2 - b^2$

(c) $\sqrt{a^2 + b^2}$ (d) $\sqrt{a^2 - b^2}$ *(2020)*

34. If 3 sin A = 1, then find the value of sec A. (2021 C)

35.

Show that : $\frac{1 + \cot^2\theta}{1 + \tan^2\theta} = \cot^2\theta$ *(2021C)*

36. 5 tan20-5 sec^2 0 = ________ (2020 C)

37. Simplest form of (1 - cos^2 A) (1 + cot^2 A) is ______ (2020)

38.

Simplest form of $\frac{1+\tan^2 A}{1+\cot^2 A}$ is _____. *(2020)*

39.

The value of $\left(\sin^2\theta+\frac{1}{1+\tan^2\theta}\right)$ = _____. *(2020)*

40. The value of (1 + $tan^2$0)(1 - sine)(1 + sine) (2020)

41. If $cosec^2$ 0 (1 + cos 0)(1 − cos 0) = k, then find the value of k. (2019 C)

42. If seco+tan0 = x, find the value of seco - tano. (Board Term 1, 2017)

43. Find the value of ($sec^2$0 - 1).$cot^2$0 (Board Term 1, 2017)

44. Write the expression in simplest form:

$\sec^2\theta-\frac{1}{\text{cosec}^2\theta-1}$. *(Board Term I, 2016)*

SAI (2 marks)

45. If sine+cose=√3, then find the value of sine cose. (2023)

46.

If $\sin\alpha=\frac{1}{\sqrt{2}}$ and $\cot\beta=\sqrt{3}$, then find the value of $\text{cosec}\,\alpha+\text{cosec}\,\beta$. *(2023)*

47. If x = p seco + q tane and y = p tano + q seco, then prove that x^2 - y^2 = p^2 - q^2. (Board Term I, 2017)

48.

Prove that :

$\frac{1+\tan^2 A}{1+\cot^2 A}=\tan^2 A.$ *(Board Term I, 2016)*

49.

Prove that : $\sqrt{\frac{1+\cos A}{1-\cos A}} = \operatorname{cosec} A + \cot A.$

(Board Term I, 2015)

SA II (3 marks)

50. Prove that:

$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$

(2023, 2018, Board Term I, 2016)

51. Prove that sec A (1 - sin A) (sec A + tan A)= 1. (2023)

52. Prove that

$(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\cot A + \tan A}.$

(NCERT, 2023)

53. Show that $\sin^6 A + 3\sin^2 A \cos^2 A = 1 - \cos^6 A$ (2021 C)

54.

Prove that $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \frac{1-\sin\theta}{\cos\theta}$ *(2020 C)*

55.

Show that $\frac{1+\tan A}{2\sin A} + \frac{1+\cot A}{2\cos A} = \operatorname{cosec} A + \sec A$

(2020 C)

56.

Prove that : $\frac{2\cos^3\theta - \cos\theta}{\sin\theta - 2\sin^3\theta} = \cot\theta$ *(2020)*

57. Prove that:

$(\sin^4 0 - \cos^4 0 + 1) \operatorname{cosec}^2 0 = 2$ (2020)

58.

Prove that : $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$ *(2020)*

59. If sine+cose=√3, then prove that tane + coto = 1. (2020)

60.

Prove that $1+\frac{\cot^2\theta}{1+\operatorname{cosec}\theta}=\operatorname{cosec}\theta$ *(2019 C)*

61. Prove that (sine + coseco)2 + (cose + seco)^2
= 7+ tan^{2}0+ cot^{2}0. (Delhi 2019, Board Term I, 2015)

62. Prove that
(1 + cotA - cosecA)(1 + tanA + secA) = 2. (Delhi 2019)

63.

Prove that :

$\frac{\tan\theta}{1-\tan\theta}-\frac{\cot\theta}{1-\cot\theta}=\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$ *(AI 2019)*

64. If cose+sine=√2 cose, show that
cose-sine = √2 sine. (AI 2019)

65.

If 4 tan θ = 3, evaluate $\left(\frac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}\right)$ *(2018)*

66.

Prove that : $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$

(Board Term I, 2017, 2015)

67.

If $\tan A=\frac{1}{2}$, find the value of

$\frac{\cos A}{\sin A}+\frac{\sin A}{1+\cos A}$. *(Board Term I, 2017)*

68.

Prove that :

$\frac{\operatorname{cosec}A-\sin A}{\operatorname{cosec}A+\sin A}=\frac{\sec^2 A-\tan^2 A}{\sec^2 A+\tan^2 A}$ *(Board Term I, 2017)*

69.

If $\sin\theta = \frac{12}{13}$, $0° < \theta < 90°$, find the value of

$$\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta \cdot \cos\theta} \times \frac{1}{\tan^2\theta}.$$ *(Board Term I, 2017)*

70. Prove that : sin20-tane + cos20.cote+2sine.cos0
= tane + cote. (Board Term 1, 2017)

71.

Prove the identity :

$$\frac{1}{\text{cosec}\theta + \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\text{cosec}\theta - \cot\theta}$$

(Board Term I, 2017)

LA (4/5/6 marks)

72. If 1 + $\sin^2$ 0 = 3 sin 0 cos 0 then prove that tan 0 = 1

or $\tan\theta = \frac{1}{2}$ *(2019)*

73.

Prove that

$$\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A} = \frac{1}{1 - 2\cos^2 A}.$$ *(Delhi 2019)*

74. Express sinA, cosA, cosecA and secA in terms of cotA. (Board Term 1, 2017)

75. If sinA + $\sin^3$A = cos2A, prove that
cos A - $4\cos^1$A + $8\cos^2$A = 4 (Board Term 1, 2017)

76. Prove that (cotA + secB)2 - (tan - cosecA)2
= 2(cotA•secB + tanB-cosecA) (Board Term I, 2017)

7.

If secA − tanA = x, show that $\frac{x^2+1}{x^2-1} = -\text{cosec}A.$

(Board Term I, 2017)

78.

Prove that : $\frac{\operatorname{cosec}A - \cot A}{\operatorname{cosec}A + \cot A} + \frac{\operatorname{cosec}A + \cot A}{\operatorname{cosec}A - \cot A}$

$= 2(2\operatorname{cosec}^2 A - 1) = 2\left(\frac{1+\cos^2 A}{1-\cos^2 A}\right)$ *(Board Term I, 2017)*

79.

If $m = \cos A - \sin A$ and $n = \cos A + \sin A$, then show that

$\frac{m}{n} - \frac{n}{m} = -\frac{4\sin A\cos A}{\cos^2 A - \sin^2 A} = -\frac{4}{\cot A - \tan A}$

(Board Term I, 2017) Ap

80.

Prove that : $\frac{\sec^3\theta}{\sec^2\theta - 1} + \frac{\operatorname{cosec}^3\theta}{\operatorname{cosec}^2\theta - 1}$

$= \sec\theta \operatorname{cosec}\theta(\sec\theta + \operatorname{cosec}\theta)$ *(Board Term I, 2017)*

81.

Prove that :

$(\tan\theta + \sec\theta - 1)\cdot(\tan\theta + 1 + \sec\theta) = \frac{2\sin\theta}{1-\sin\theta}$

(Board Term I, 2016)

82.

Prove that : $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = (\tan\theta + \cot\theta)$

(Board Term I, 2016)

83. If tanesine = m and tane sine = n; prove that:
$m^2 - n^2 = 4\sqrt{mn}$. (Board Term 1, 2015)

CBSE Sample Questions

8.2 Trigonometric Ratios

MCQ

1.

If $5\tan\beta = 4$, then $\frac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta} =$

(a) 1/3 (b) 2/5 (c) 3/5 (d) 6

(2022-23)

2.

If $4\tan\beta = 3$, then $\frac{4\sin\beta - 3\cos\beta}{4\sin\beta + 3\cos\beta} =$

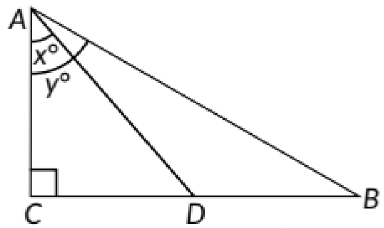
(a) 0 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

(Term I, 2021-22)

3. If $\tan a + \cot a = 2$, then $\tan^{20}a + \cot^{20}a =$
(a) 0
(b) 2
(c) 20
(d) 220 (Term I, 2021-22)

4.

In the given figure, D is the mid-point of BC, then the value of $\frac{\cot y°}{\cot x°}$ is



(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

(Term I, 2021-22) Ap

SAI (2 marks)

5. If tan A = 3/4, find the value of 1/sin A + 1/cos A. (2020-21)

8.3 Trigonometric Ratios of Some Specific Angles

MCQ

6. If x tan 60°cos 60° = sin 60°cot 60°, then x =
(a) cos 30°
(b) tan 30°
(c) sin 30°
(d) cot 30° (2022-23)

7. In AABC right angled at B, if tanA=√3, then
cosA cosC - sinAsinC =

(a) -1 (b) 0 (c) 1 (d) $\frac{\sqrt{3}}{2}$

(Term I, 2021-22)

8. If the angles of AABC are in the ratio 1:1:2, respectively (the largest angle being angle C), then
the value of $\frac{\sec A}{\operatorname{cosec} B} - \frac{\tan A}{\cot B}$ is

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

(Term I, 2021-22)

VSA (1 mark)

9. sin A + cos B = 1, A = 30° and B is an acute angle, then find the value of B. (2020-21)

SAI (2 marks)

10.

If $\sin(A + B) = 1$ and $\cos(A - B) = \frac{\sqrt{3}}{2}$, $0° < A + B \leq 90°$ and $A > B$, then find the measures of angles A and B.

(2022-23)

11.

Find an acute angle θ when $\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{1-\sqrt{3}}{1+\sqrt{3}}$.

(2022-23)

12. If $\sqrt{3}$ sin 0 - cos 0 = 0 and 0° < 0 < 90°, find the value of 0. (2020-21)

8.4 Trigonometric Identities

MCQ

13. If sine + cose =
$\sqrt{2}$, then tane + cot 0 =
(a) 1
(b) 2
(c) 3
(d) 4 (2022-23)

14. If $2\sin^2$ß - $\cos^2$ß = 2, then ß is
(a) 0°
(b) 90°
(c) 45°
(d) 30° (Term I, 2021-22)

15. If 1 + $\sin^2\alpha$ = 3sina cosa, then values of cota are
(a) -1,1
(b) 0,1
(c) 1,2
(d) -1,-1 (Term I, 2021-22)

VSA (1 mark)

16. If x = 2 sin20 and y = 2 $\cos^2$ 0 + 1, then find x + y. (2020-21)

SA II (3 marks)

17.

Prove that :

$$\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\operatorname{cosec}\theta-2\sin\theta\cos\theta$$

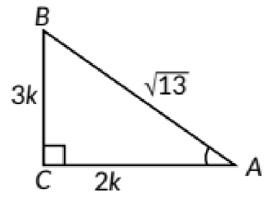
(2022-23)

SOLUTIONS

Previous Years' CBSE Board Questions

1.

(c) : We have, $2\tan A = 3$



$\Rightarrow \quad \tan A = \frac{3}{2} = \frac{P}{B}$

Let $P = 3k$ and $B = 2k$

$AB = \sqrt{2^2 + 3^2}$

(By Pythagoras theorem)

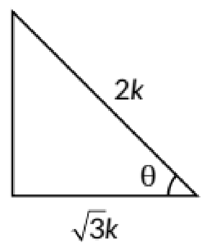
$\Rightarrow \quad H = \sqrt{13}$

$\therefore \quad \sin A = \frac{P}{H} = \frac{3}{\sqrt{13}}, \cos A = \frac{B}{H} = \frac{2}{\sqrt{13}}$

Now, $\frac{4\sin A + 3\cos A}{4\sin A - 3\cos A} = \frac{4\left(\frac{3}{\sqrt{13}}\right) + 3\left(\frac{2}{\sqrt{13}}\right)}{4\left(\frac{3}{\sqrt{13}}\right) - 3\left(\frac{2}{\sqrt{13}}\right)} = 3$

2.

(c) : Given, $\cos\theta = \frac{\sqrt{3}}{2} = \frac{B}{H}$



Let $B = \sqrt{3}k$ and $H = 2k$

$\therefore \quad P = \sqrt{(2k)^2 - (\sqrt{3}k)^2}$

[By Pythagoras Theorem]

$\Rightarrow \quad P = \sqrt{k^2} = k$

$\therefore \quad \text{cosec}\,\theta = \frac{H}{P} = \frac{2k}{k} = 2 \qquad \sec\theta = \frac{H}{B} = \frac{2k}{\sqrt{3}k} = \frac{2}{\sqrt{3}}$

$\therefore \quad \frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{(2)^2 - \left(\frac{2}{\sqrt{3}}\right)^2}{(2)^2 + \left(\frac{2}{\sqrt{3}}\right)^2} = \frac{4 - \frac{4}{3}}{4 + \frac{4}{3}} = \frac{8}{16} = \frac{1}{2}$

3.

(c) : We have, $\frac{1}{\operatorname{cosec}\theta(1-\cot\theta)}+\frac{1}{\sec\theta(1-\tan\theta)}$

$$=\frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}$$

$$\left[\because \frac{1}{\operatorname{cosec}\theta}=\sin\theta,\frac{1}{\sec\theta}=\cos\theta,\tan\theta=\frac{\sin\theta}{\cos\theta},\cot\theta=\frac{\cos\theta}{\sin\theta}\right]$$

$$=\frac{\sin^2\theta}{\sin\theta-\cos\theta}+\frac{\cos^2\theta}{\cos\theta-\sin\theta}=\frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}=\sin\theta+\cos\theta$$

4.

(a): We have $\sin\theta = \cos\theta$

or $\frac{\sin\theta}{\cos\theta}=1$

$\Rightarrow$ $\tan\theta = 1$ and $\cot\theta = 1$ $\quad\left[\because \cot\theta=\frac{1}{\tan\theta}\right]$

$\therefore$ $\tan^2\theta + \cot^2\theta = 1^2 + 1^2 = 2$

Hence, A option is correct.

5.

We have $\tan\theta+\cot\theta=\frac{4\sqrt{3}}{3}$ ...(i)

On squaring both sides of equation (i), we get

$$\tan^2\theta+\cot^2\theta+2\tan\theta\cdot\cot\theta=\frac{16\times3}{9}$$

$$\Rightarrow \tan^2\theta+\cot^2\theta+2\tan\theta\cdot\frac{1}{\tan\theta}=\frac{16}{3}$$

$$\Rightarrow \tan^2\theta+\cot^2\theta=\frac{16}{3}-2=\frac{10}{3}$$

6.

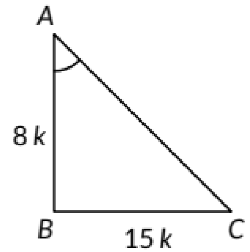
In right angle ΔABC, we have

$15 \cot A = 8$

$\Rightarrow \cot A = \frac{8}{15}$

Since, $\cot A = \frac{AB}{BC}$

$\therefore \frac{AB}{BC} = \frac{8}{15}$



Let $AB = 8k$ and $BC = 15k$

By using Pythagoras theorem, we get

$AC^2 = AB^2 + BC^2$

$\Rightarrow (8k)^2 + (15k)^2 = 64k^2 + 225k^2 = 289k^2 = (17k)^2$

$\Rightarrow AC = \sqrt{(17k)^2} = 17k$

$\therefore \sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$ and $\cos A = \frac{AB}{AC} = \frac{8k}{17k} = \frac{8}{17}$

So, $\sec A = \frac{1}{\cos A} = \frac{17}{8}$

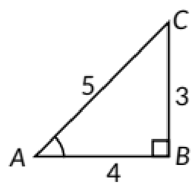
7.

Given, $3 \cot A = 4 \Rightarrow \cot A = \frac{4}{3} \Rightarrow \tan A = \frac{3}{4}$

In ΔABC, $AC^2 = AB^2 + BC^2 = 16 + 9 = 25 \Rightarrow AC = 5$

Now, L.H.S. = $\frac{1 - \tan^2 A}{1 + \tan^2 A}$

$= \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$



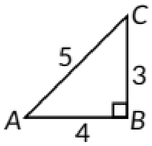
R.H.S. = $\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$\therefore$ L.H.S. = R.H.S.

8.

We have, $\sin A = \frac{3}{5} = \frac{P}{H}$

In right angled ΔABC, by Pythagoras theorem, we have



$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad 5^2 = AB^2 + (3)^2$

$\Rightarrow \quad AB^2 = 16 \Rightarrow AB = 4$

$\therefore \quad \cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{4}{5}, \tan A = \frac{P}{B} = \frac{3}{4}, \operatorname{cosec} A = \frac{5}{3},$

$\sec A = \frac{5}{4}$ and $\cot A = \frac{4}{3}$

9.

We have, $3 \tan A = 4$

$\Rightarrow \quad \tan A = \frac{4}{3} = \frac{\text{Perpendicular}}{\text{Base}}$

$\therefore \quad$ Hypotenuse $= \sqrt{(4)^2 + (3)^2} = 5$

$\therefore \quad \sin A = \frac{4}{5}$ and $\cos A = \frac{3}{5}$

Now, L.H.S. $= \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{4}{3}\right)^2}{1 + \left(\frac{4}{3}\right)^2} = \frac{1 - \frac{16}{9}}{1 + \frac{16}{9}} = -\frac{7}{25}$

and R.H.S. $= \cos^2 A - \sin^2 A$

$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$

$\therefore \quad$ L.H.S. = R.H.S.

10.

$= \frac{5}{8} \times (2)^2 - (\sqrt{3})^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{5}{8} \times 4 - 3 + \frac{1}{2} = 0$

11.

(a): Given, $\sin\alpha = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \alpha = 60°$ $\quad \left(\because \quad \sin 60° = \frac{\sqrt{3}}{2}\right)$

and $\tan\beta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \beta = 30°$ $\quad \left(\because \quad \tan 30° = \frac{1}{\sqrt{3}}\right)$

Now, $\cos(\alpha - \beta) = \cos(60° - 30°) = \cos 30° = \frac{\sqrt{3}}{2}$

12.

(a): Given, $2 \sin 2\theta = 1 \Rightarrow \sin 2\theta = 1/2$

$\Rightarrow \quad 2\theta = 30°$ $\quad \left(\because \quad \sin 30° = \frac{1}{2}\right)$

$\Rightarrow \quad \theta = 15°$

13.

We have, $2 \sec 30° \times \tan 60°$

$= 2 \times \frac{2}{\sqrt{3}} \times \sqrt{3} = 4$

14.

We have, $\sin^2 30° + \cos^2 60°$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$

15.

We have, $\frac{2\tan 45° \times \cos 60°}{\sin 30°} = \frac{2 \times 1 \times \frac{1}{2}}{\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$

16.

Given, $\sin x + \cos y = 1$

$\Rightarrow \sin 30° + \cos y = 1$ [Given, $x = 30°$]

$\Rightarrow \frac{1}{2} + \cos y = 1 \Rightarrow \cos y = 1 - \frac{1}{2} = \frac{1}{2}$

$\Rightarrow \cos y = \cos 60° \Rightarrow y = 60°$

17.

We have, $\sin\alpha = \frac{1}{2}$

Now, $3\sin\alpha - 4\sin^3\alpha = 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3 = \frac{3}{2} - \frac{4}{8} = \frac{3}{2} - \frac{1}{2} = 1$

18.

Put $\theta = 45°$ in $2\sec^2\theta + 3\,\text{cosec}^2\,\theta - 2\sin\theta\cos\theta$

$= 2\sec^2(45°) + 3\,\text{cosec}^2\,(45°) - 2\sin(45°)\cos(45°)$

$= 2(\sqrt{2})^2 + 3(\sqrt{2})^2 - 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 4 + 6 - 1 = 9$

19.

Given, $\sin\theta - \cos\theta = 0$

$\Rightarrow \sin\theta = \cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \tan\theta = 1$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$\Rightarrow \tan\theta = \tan 45°$ $[\because \tan 45° = 1]$

$\Rightarrow \theta = 45°$

$\therefore \sin^4\theta + \cos^4\theta = \sin^4(45°) + \cos^4(45°)$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = 2\left(\frac{1}{\sqrt{2}}\right)^4 = 2\left(\frac{1}{4}\right) = \frac{1}{2}$

20.

$\frac{5}{\cot^2 30°} + \frac{1}{\sin^2 60°} - \cot^2 45° + 2\sin^2 90°$

$= \frac{5}{(\sqrt{3})^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - (1)^2 + 2(1)^2 = \frac{5}{3} + \frac{4}{3} - 1 + 2 = 4$

21.

Given, $\sin\theta = \cos\theta$

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = 1 \Rightarrow \tan\theta = 1$

$\Rightarrow \quad \tan\theta = \tan\frac{\pi}{4}$ [θ is acute]

$\therefore \quad \theta = \frac{\pi}{4}$

So, $\tan^2\theta + \cot^2\theta - 2$

$= \tan^2\left(\frac{\pi}{4}\right) + \cot^2\left(\frac{\pi}{4}\right) - 2 = 1 + 1 - 2 = 0$

22.

Given that, $A = 60°$, $B = 30°$

$\therefore \quad \cos A = \cos 60° = \frac{1}{2}$; $\cos B = \cos 30° = \frac{\sqrt{3}}{2}$

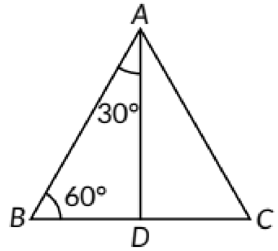
Now, $\cos A + \cos B = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

and $\cos(A + B) = \cos(60° + 30°) = \cos 90° = 0$

$\therefore \quad \cos(A + B) \neq \cos A + \cos B$.

23.

Consider an equilateral triangle *ABC* with each side of length $2a$, and $\angle A = \angle B = \angle C = 60°$



$\Rightarrow \quad AB = BC = CA = 2a$

Now, draw $AD \perp BC$

Now, in ΔADB and ΔADC

$\angle ADB = \angle ADC$ (each 90°)

$AB = AC$

$AD = AD$ (common)

$\therefore \quad \Delta ADB \cong \Delta ADC$ (By R.H.S.)

$\therefore \quad BD = DC$ and $\angle BAD = \angle CAD$ (By C.P.C.T.)

$\therefore \quad BD = DC = a$ and $\angle BAD = 30°$

In ΔADB, $AB = 2a$, $BD = a$, $\angle DAB = 30°$

$\therefore \quad \operatorname{cosec} 30° = \frac{AB}{BD} = \frac{2a}{a} = 2$

Again in ΔADB, we have $\angle ABD = 60°$

$\therefore \quad \cos 60° = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$

24.

We have, $\sin(A + B) = 1$

$\Rightarrow \quad \sin(A + B) = \sin 90° \Rightarrow A + B = 90°$ ...(i)

Also, $\sin(A - B) = \frac{1}{2}$

$\Rightarrow \quad \sin(A - B) = \sin 30° \Rightarrow A - B = 30°$ ...(ii)

Adding (i) and (ii), we get

$\Rightarrow \quad (A + B) + (A - B) = 120° \Rightarrow 2A = 120° \Rightarrow A = 60°$

From (i), we have $60° + B = 90° \Rightarrow B = 30°$

25.

Given, $\theta = 30°$

(i) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Put $\theta = 30°$, we get

R.H.S. $= 4\cos^3 30° - 3\cos 30°$

$= 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3\left(\frac{\sqrt{3}}{2}\right) = \frac{4\times 3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0 = \cos 90°$

$= \cos(3 \times 30°) = \cos 3\theta =$ L.H.S.

(ii) R.H.S. $= 3\sin\theta - 4\sin^3\theta$

$= 3\sin 30° - 4\sin^3 30° = 3\times\frac{1}{2} - 4\times\frac{1}{8} = \frac{3}{2} - \frac{1}{2} = 1$

L.H.S. $= \sin 3\theta = \sin(3 \times 30°) = \sin 90° = 1 \quad \therefore$ L.H.S. = R.H.S.

26.

We know that, $\sin 30° = \frac{1}{2}$; $\cos 30° = \frac{\sqrt{3}}{2}$

$$\tan 30° = \frac{\sin 30°}{\cos 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}\ ;\ \cot 30° = \frac{1}{\tan 30°} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$

$$\sec 30° = \frac{1}{\cos 30°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}\ ;\ \operatorname{cosec} 30° = \frac{1}{\sin 30°} = \frac{1}{\frac{1}{2}} = 2$$

$$\sin 45° = \frac{1}{\sqrt{2}}\ ;\ \cos 45° = \frac{1}{\sqrt{2}}\ ;\ \tan 45° = \frac{\sin 45°}{\cos 45°} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

$$\cot 45° = \frac{1}{\tan 45°} = 1\ ;\ \sec 45° = \frac{1}{\cos 45°} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\operatorname{cosec} 45° = \frac{1}{\sin 45°} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

27.

Given, $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

(i) Putting $A = 45°, B = 30°$, we get,

$\sin(45° + 30°) = \sin 45° \cos 30° + \cos 45° \sin 30°$

$$\Rightarrow \sin 75° = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) $\cos(45° - 30°) = \cos 45° \cos 30° + \sin 45° \sin 30°$

$$\Rightarrow \cos 15° = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

28.

(b): We have, $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$

$= (\tan^2\theta)(\cot^2\theta)$

$(\because \sec^2\theta - 1 = \tan^2\theta, \operatorname{cosec}^2\theta - 1 = \cot^2\theta)$

$$= \tan^2\theta \times \frac{1}{\tan^2\theta} = 1 \qquad \left(\because \cot\theta = \frac{1}{\tan\theta}\right)$$

29. (c): $\sec^2 0 - \tan^2 0 = 1$

30.

(c): Given, $\sin\theta = \frac{p}{q}$

$$\therefore \quad \cos\theta = \sqrt{1 - \left(\frac{p}{q}\right)^2} \qquad \left[\because \cos\theta = \sqrt{1 - \sin^2\theta}\right]$$

$$\Rightarrow \quad \cos\theta = \frac{\sqrt{q^2 - p^2}}{q}$$

$$\text{Now, } \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{p}{q}}{\frac{\sqrt{q^2 - p^2}}{q}} = \frac{p}{\sqrt{q^2 - p^2}}$$

31.

(d): $\sqrt{(1 - \cos^2\theta)(1 + \tan^2\theta)} = \sqrt{\sin^2\theta \cdot \sec^2\theta}$

$[\because \ 1 + \tan^2\theta = \sec^2\theta]$

$$= \sqrt{\frac{\sin^2\theta}{\cos^2\theta}} \qquad \left[\because \sec^2\theta = \frac{1}{\cos^2\theta}\right]$$

$$= \sqrt{\tan^2\theta} = \tan\theta \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

32. (b): Given, sin^{2}0 + sine = 1 ...(i)
⇒ sin0 = 1-sin^{2}0 ⇒ sine = cos^{2}0 ...(ii)
:- cos^{2}0 + cos^{1}0
= sine + sin^{2}0 [From (ii)]
= 1 [From (i)]

33. (c): Let A(acose + bsin0, 0) and B(O, asino - bcose) Using distance formula, we have

$$AB = \sqrt{(a\cos\theta + b\sin\theta - 0)^2 + (0 - a\sin\theta + b\cos\theta)^2}$$

$$= \sqrt{\begin{aligned}&a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta \\ &+ a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta\end{aligned}}$$

$$= \sqrt{a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)}$$

$$= \sqrt{a^2 + b^2} \qquad (\because \sin^2\theta + \cos^2\theta = 1)$$

34.

We have 3 sin A = 1

$\therefore \sin A = \frac{1}{3}$

Now by using $\cos^2 A = 1 - \sin^2 A$, we get

$$\cos^2 A = 1 - \frac{1}{9} = \frac{8}{9} \Rightarrow \cos A = \frac{2\sqrt{2}}{3}$$

$$\therefore \sec A = \frac{1}{\cos A} = \frac{1}{\frac{2\sqrt{2}}{3}} = \frac{3}{2\sqrt{2}}$$

35.

We have L.H.S.

$$\frac{1+\cot^2\theta}{1+\tan^2\theta} = \frac{\mathrm{cosec}^2\theta}{\sec^2\theta}$$

[By using $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \mathrm{cosec}^2\theta$]

$$\Rightarrow \frac{1/\sin^2\theta}{1/\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta = \text{R.H.S.}$$

Hence, $\frac{1+\cot^2\theta}{1+\tan^2\theta} = \cot^2\theta$

36. We have, $5(\tan^2 0 - \sec^2 0)$

$=5(-1) = -5$

{By using $1 + \tan^2 0 = \sec^2 0 \Rightarrow \tan^2 0 - \sec^2 0 = -1$}

37.

$(1 - \cos^2 A)(1 + \cot^2 A)$

$$= (1-\cos^2 A)\left(1+\frac{\cos^2 A}{\sin^2 A}\right) \qquad \left(\because \cot A = \frac{\cos A}{\sin A}\right)$$

$$= (1-\cos^2 A)\left(\frac{\sin^2 A+\cos^2 A}{\sin^2 A}\right)$$

$$= \frac{\sin^2 A}{\sin^2 A} \quad (\because \sin^2 A + \cos^2 A = 1) = 1$$

38.

We have,

$$\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{\sec^2 A}{\mathrm{cosec}^2 A}=\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}=\frac{\sin^2 A}{\cos^2 A}=\tan^2 A$$

39.

We have, $\sin^2\theta+\frac{1}{1+\tan^2\theta}$

$=\sin^2\theta+\frac{1}{\sec^2\theta}$ [$\because 1+\tan^2\theta=\sec^2\theta$]

$= \sin^2\theta + \cos^2\theta$ [$\because \sec\theta = 1/\cos\theta$]

$= 1$ [$\because \sin^2\theta + \cos^2\theta = 1$]

40.

We have, $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$= \sec^2\theta\,(1 - \sin^2\theta) = \sec^2\theta\cos^2\theta = \frac{1}{\cos^2\theta} \times \cos^2\theta$

$= 1$ $\left[\sec\theta=\frac{1}{\cos\theta}\right]$

41.

We have $\mathrm{cosec}^2\,\theta\,(1 + \cos\theta)(1 - \cos\theta) = k$

[$\because (a + b)(a - b) = (a^2 - b^2)$]

$\Rightarrow \mathrm{cosec}^2\,\theta\,(1 - \cos^2\theta) = k$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow \mathrm{cosec}^2\,\theta\,(\sin^2\theta) = k$

$\Rightarrow \frac{1}{\sin^2\theta}\cdot\sin^2\theta=k$

$\Rightarrow k = 1$

42.

Given, $\sec\theta + \tan\theta = x$

Now, we know that, $1 = \sec^2\theta - \tan^2\theta$

$\Rightarrow \quad 1 = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$\Rightarrow \quad 1 = x(\sec\theta - \tan\theta)$

$\Rightarrow \quad \frac{1}{x} = \sec\theta - \tan\theta \quad \therefore \quad \sec\theta - \tan\theta = \frac{1}{x}$

43.

We have, $(\sec^2\theta - 1) \times \cot^2\theta$

$$= \left(\frac{1}{\cos^2\theta} - 1\right) \cdot \frac{\cos^2\theta}{\sin^2\theta} = \frac{1-\cos^2\theta}{\cos^2\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta} = 1$$

44.

We have, $\sec^2\theta - \frac{1}{\mathrm{cosec}^2\theta - 1}$

$$= \frac{1}{\cos^2\theta} - \frac{1}{\frac{1}{\sin^2\theta} - 1} = \frac{1}{\cos^2\theta} - \frac{1}{\frac{1-\sin^2\theta}{\sin^2\theta}}$$

$$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\sin^2\theta}{\cos^2\theta} \qquad [\because \cos^2\theta + \sin^2\theta = 1]$$

$= \frac{\cos^2\theta}{\cos^2\theta} = 1$, which is the simplest form.

45. (a) Given, sine + cose = √3

Squaring both sides, we get (sine + cose)^2 = 3

= sin20+ cos20 + 2sine cose = 3

= 2sine cose = 3-1 (:- sin20+ cos20 = 1)

= 2sine cose = 2

= sine cose = 1

46.

Given, $\sin\alpha = \frac{1}{\sqrt{2}}$ and $\cot\beta = \sqrt{3}$

We know that, $\operatorname{cosec}\alpha = \frac{1}{\sin\alpha} = \sqrt{2}$

Also, $1 + \cot^2\beta = \operatorname{cosec}^2\beta$

$\Rightarrow \quad \operatorname{cosec}^2\beta = 4$

$\Rightarrow \quad \operatorname{cosec}\beta = 2$

Now, $\operatorname{cosec}\alpha + \operatorname{cosec}\beta = \sqrt{2} + 2$

47.

We have, $x = p\sec\theta + q\tan\theta$ and $y = p\tan\theta + q\sec\theta$

Now, L.H.S. $= x^2 - y^2$

$= (p\sec\theta + q\tan\theta)^2 - (p\tan\theta + q\sec\theta)^2$

$= (p^2\sec^2\theta + q^2\tan^2\theta + 2pq\sec\theta\tan\theta)$

$\quad - (p^2\tan^2\theta + q^2\sec^2\theta + 2pq\tan\theta\sec\theta)$

$= p^2\sec^2\theta + q^2\tan^2\theta - p^2\tan^2\theta - q^2\sec^2\theta$

$= p^2(\sec^2\theta - \tan^2\theta) - q^2(\sec^2\theta - \tan^2\theta)$

$= p^2 - q^2 \qquad [\because \sec^2\theta - \tan^2\theta = 1]$

= R.H.S.

48.

We have, L.H.S. $= \frac{1+\tan^2 A}{1+\cot^2 A}$

$= \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A =$ R.H.S.

49.

$$\text{L.H.S.} = \sqrt{\frac{1+\cos A}{1-\cos A}} = \sqrt{\frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}} = \sqrt{\left(\frac{1+\cos A}{\sin A}\right)^2} = \frac{1+\cos A}{\sin A}$$

$$= \frac{1}{\sin A} + \frac{\cos A}{\sin A} = \operatorname{cosec} A + \cot A = \text{R.H.S.}$$

50.

$$\text{We have, L.H.S.} = \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$$

$$= \frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A - 1)} = \frac{\sin A(1-2(1-\cos^2 A))}{\cos A(2\cos^2 A-1)}$$

$$= \frac{\sin A(2\cos^2 A-1)}{\cos A(2\cos^2 A-1)} = \tan A = \text{R.H.S.}$$

51.

$$\text{L.H.S.} = \sec A\,(1 - \sin A)(\sec A + \tan A)$$

$$= \frac{(1-\sin A)(1+\sin A)}{\cos^2 A}$$

$$= \frac{1-\sin^2 A}{\cos^2 A} \qquad (\because\ (a-b)(a+b) = a^2 - b^2)$$

$$= \frac{\cos^2 A}{\cos^2 A} \qquad (\because\ 1 - \sin^2 A = \cos^2 A)$$

$$= 1 = \text{R.H.S.}$$

52.

L.H.S. = (cosec A – sin A) (sec A – cos A)

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right) = \frac{\cos^2 A \times \sin^2 A}{\sin A \cos A}$$

[$\because$ $1 - \sin^2 A = \cos^2 A$ and $1 - \cos^2 A = \sin^2 A$]

$$= \frac{\sin A \cdot \cos A}{1} = \frac{\sin A \cdot \cos A}{\sin^2 A + \cos^2 A}$$ [$\because$ $1 = \sin^2 A + \cos^2 A$]

$$= \frac{\frac{\sin A \cos A}{\sin A \cos A}}{\frac{\sin^2 A}{\sin A \cos A} + \frac{\cos^2 A}{\sin A \cos A}}$$ [Dividing numerator and denominator by sin A cos A]

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\tan A + \cot A} = \text{R.H.S.}$$

53.

We have $\sin^6 A + 3 \sin^2 A \cos^2 A = 1 - \cos^6 A$

Rewriting and arranging the given equation as

$\sin^6 A + \cos^6 A = 1 - 3 \sin^2 A \cos^2 A$ ...(i)

Now taking L.H.S. of equation (i), we get

$\sin^6 A + \cos^6 A = (\sin^2 A)^3 + (\cos^2 A)^3$

{By using $(a + b)^3 = a^3 + b^3 + 3ab (a + b) \Rightarrow a^3 + b^3 = (a + b)^3 - 3ab (a + b)$, here $a = \sin^2 A$ and $b = \cos^2 A$}

$\therefore$ $\sin^6 A + \cos^6 A = (\sin^2 A + \cos^2 A)^3 - 3 \sin^2 A \cos^2 A (\sin^2 A + \cos^2 A)$

$= 1^2 - 3 \sin^2 \cos A (1) =$ R.H.S. [$\because$ $\sin^2 A + \cos^2 A = 1$]

$\Rightarrow$ L.H.S. = R.H.S

Hence proved.

54.

We have, $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}=\frac{1-\sin\theta}{\cos\theta}$ ...(i)

On taking L.H.S. of equation (i), we get

$$\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}=\frac{(\sec^2\theta-\tan^2\theta)+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$$

$[\because\ 1+\tan^2\theta=\sec^2\theta]$

$$=\frac{(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)+(\sec\theta-\tan\theta)}{1+\tan\theta+\sec\theta}$$

$[\because\ (a-b)(a+b)=a^2-b^2]$

$$=\frac{(\sec\theta-\tan\theta)[(\sec\theta+\tan\theta+1)]}{[\sec\theta+\tan\theta+1]}=\sec\theta-\tan\theta$$

$$=\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}=\frac{1-\sin\theta}{\cos\theta}=\text{R.H.S.}$$

So, $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}=\frac{1-\sin\theta}{\cos\theta}$

Hence proved.

55.

We have, $\frac{1+\tan A}{2\sin A}+\frac{1+\cot A}{2\cos A}=\text{cosec}\, A+\sec A$ ...(i)

On taking L.H.S. of equation (i), we get

$$\Rightarrow \frac{1+\frac{\sin A}{\cos A}}{2\sin A}+\frac{1+\frac{\cos A}{\sin A}}{2\cos A}$$

$$\Rightarrow \frac{\cos A+\sin A}{2\sin A\cos A}+\frac{\sin A+\cos A}{2\sin A\cos A}$$

$$=\frac{\cos A+\sin A+\sin A+\cos A}{2\sin A\cos A}=\frac{2[\cos A+\sin A]}{2\sin A\cos A}$$

$$=\frac{\sin A+\cos A}{\sin A\cos A}=\frac{\sin A}{\sin A\cos A}+\frac{\cos A}{\sin A\cos A}$$

$$=\frac{1}{\cos A}+\frac{1}{\sin A}=\sec A+\text{cosec}\, A=\text{R.H.S.}$$

Hence, $\frac{1+\tan A}{2\sin A}+\frac{1+\cot A}{2\cos A}=\text{cosec}\, A+\sec A$ proved.

56.

$$\text{L.H.S.} = \frac{2\cos^3\theta-\cos\theta}{\sin\theta-2\sin^3\theta}$$

$$=\frac{\cos\theta(2\cos^2\theta-1)}{\sin\theta(1-2\sin^2\theta)}=\frac{\cot\theta(2(1-\sin^2\theta)-1)}{(1-2\sin^2\theta)}$$

$(\because\ \sin^2\theta+\cos^2\theta=1)$

$$=\frac{\cot\theta(2-2\sin^2\theta-1)}{(1-2\sin^2\theta)}=\frac{\cot\theta(1-2\sin^2\theta)}{(1-2\sin^2\theta)}=\cot\theta=\text{R.H.S.}$$

57.

We know that $\sin^2\theta + \cos^2\theta = 1$

Squaring both sides, we get

$(\sin^2\theta + \cos^2\theta)^2 = 1$

$\Rightarrow \quad \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta = 1$

$\Rightarrow \quad \sin^4\theta + \cos^4\theta + 2\sin^2\theta(1 - \sin^2\theta) = 1$

$\Rightarrow \quad \sin^4\theta + \cos^4\theta + 2\sin^2\theta - 2\sin^4\theta = 1$

$\Rightarrow \quad \cos^4\theta - \sin^4\theta + 2\sin^2\theta = 1$

$\Rightarrow \quad \sin^4\theta - \cos^4\theta - 2\sin^2\theta = -1$

$\Rightarrow \quad \sin^4\theta - \cos^4\theta + 1 = 2\sin^2\theta$

$\Rightarrow \quad (\sin^4\theta - \cos^4\theta + 1)\operatorname{cosec}^2\theta = 2$

58.

$$\text{L.H.S.} = \sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \sqrt{\left(\frac{1+\sin A}{\cos A}\right)^2}$$

$$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A = \text{R.H.S.}$$

59.

Given, $\sin\theta + \cos\theta = \sqrt{3}$

Squaring both sides, we get $(\sin\theta + \cos\theta)^2 = 3$

$\Rightarrow \quad \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow \quad 2\sin\theta\cos\theta = 3 - 1 \Rightarrow 2\sin\theta\cos\theta = 2$

$\Rightarrow \quad \sin\theta\cos\theta = 1$ ...(i)

$$\text{L.H.S.} = \tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = \frac{1}{1}$$ [Using (i)]

$= 1 = \text{R.H.S.}$

60.

We have, $1+\frac{\cot^2\theta}{1+\operatorname{cosec}\theta}=\operatorname{cosec}\theta$

On taking L.H.S. of given equation, we have

$$1+\frac{\cot^2\theta}{1+\operatorname{cosec}\theta}=1+\frac{\cos^2\theta/\sin^2\theta}{1+\frac{1}{\sin\theta}}=1+\frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{\sin\theta+1}{\sin\theta}}$$

$$=1+\frac{\cos^2\theta}{\sin^2\theta+\sin\theta}$$

$$=\frac{\sin^2\theta+\sin\theta+\cos^2\theta}{\sin\theta(\sin\theta+1)}=\frac{(\sin^2\theta+\cos^2\theta)+\sin\theta}{\sin\theta(\sin\theta+1)}$$

$$=\frac{(1+\sin\theta)}{\sin\theta(\sin\theta+1)}=\frac{1}{\sin\theta}\qquad [\because\ \sin^2\theta+\cos^2\theta=1]$$

$$\left[\because\ \operatorname{cosec}\theta=\frac{1}{\sin\theta}\right]$$

$= \operatorname{cosec}\theta =$ R.H.S.

$\Rightarrow$ L.H.S. = R.H.S.

Hence proved.

61.

We have,

L.H.S. $= (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$= (\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta) + (\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta)$

$= (\sin^2\theta + \operatorname{cosec}^2\theta + 2) + (\cos^2\theta + \sec^2\theta + 2)$

$= (\sin^2\theta + \cos^2\theta) + \operatorname{cosec}^2\theta + \sec^2\theta + 4$

$= 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 4$

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 4 = 7 + \tan^2\theta + \cot^2\theta =$ R.H.S.

62.

L.H.S. = $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$

$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$

$$= \left(\frac{(\sin A + \cos A - 1)(\sin A + \cos A + 1)}{\sin A \cos A}\right) = \frac{(\sin A + \cos A)^2 - 1^2}{\sin A \cos A}$$

$$= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A - 1}{\sin A \cos A}$$

$$= \frac{1 + 2\sin A \cos A - 1}{\sin A \cos A} \qquad [\because \sin^2 A + \cos^2 A = 1]$$

= 2 = R.H.S.

63.

$$\text{L.H.S.} = \frac{\tan\theta}{1 - \tan\theta} - \frac{\cot\theta}{1 - \cot\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1 - \frac{\sin\theta}{\cos\theta}} - \frac{\frac{\cos\theta}{\sin\theta}}{1 - \frac{\cos\theta}{\sin\theta}} = \frac{\sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\sin\theta + \cos\theta}{\cos\theta - \sin\theta} = \text{R.H.S.}$$

64.

Given, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = (\sqrt{2} - 1)\cos\theta$

Multiplying both sides by $(\sqrt{2} + 1)$, we get

65.

Given, 4 tan θ = 3

$\Rightarrow \tan\theta = \frac{3}{4}$ and $\sec\theta = \sqrt{1+\tan^2\theta}$

$\Rightarrow \sec\theta = \sqrt{1+\left(\frac{3}{4}\right)^2} = \sqrt{\frac{16+9}{16}} = \frac{5}{4}$

We have, $\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \frac{4\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{4\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}}$

$= \frac{4\tan\theta - 1 + \sec\theta}{4\tan\theta + 1 - \sec\theta}$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \sec\theta = \frac{1}{\cos\theta}\right]$

$= \frac{4\left(\frac{3}{4}\right) - 1 + \frac{5}{4}}{4\left(\frac{3}{4}\right) + 1 - \frac{5}{4}} = \frac{\frac{12}{4} - 1 + \frac{5}{4}}{\frac{12}{4} + 1 - \frac{5}{4}} = \frac{\frac{12-4+5}{4}}{\frac{12+4-5}{4}} = \frac{13}{11}$

66.

We have, L.H.S. = $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

$= \frac{\frac{\sin\theta - \cos\theta + 1}{\cos\theta}}{\frac{\sin\theta + \cos\theta - 1}{\cos\theta}} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$

$= \frac{\tan\theta + \sec\theta - 1}{\tan\theta + 1 - \sec\theta} = \frac{\tan\theta + \sec\theta - [\sec^2\theta - \tan^2\theta]}{\tan\theta + 1 - \sec\theta}$

$= \frac{\tan\theta + \sec\theta - [(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)]}{\tan\theta + 1 - \sec\theta}$

$= \frac{(\tan\theta + \sec\theta)(1 - \sec\theta + \tan\theta)}{1 - \sec\theta + \tan\theta}$

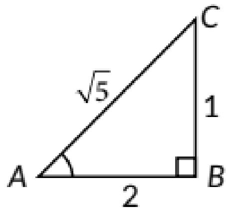
$= (\tan\theta + \sec\theta) = (\tan\theta + \sec\theta) \times \frac{\tan\theta - \sec\theta}{\tan\theta - \sec\theta}$

$= \frac{\tan^2\theta - \sec^2\theta}{\tan\theta - \sec\theta} = \frac{-1}{\tan\theta - \sec\theta} = \frac{1}{\sec\theta - \tan\theta}$ = R.H.S.

67.

Given, $\tan A = \frac{1}{2}$

Consider a right angled ΔABC,



By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC^2 = 4 + 1 \Rightarrow AC = \sqrt{5}$

Now,

$$\frac{\cos A}{\sin A} + \frac{\sin A}{1+\cos A} = \frac{\cos A(1+\cos A)+\sin^2 A}{\sin A(1+\cos A)}$$

$$= \frac{\cos A+(\cos^2 A+\sin^2 A)}{\sin A(1+\cos A)} = \frac{\cos A+1}{\sin A(1+\cos A)}$$

$$= \frac{1}{\sin A} = \operatorname{cosec} A = \sqrt{5}.$$

68.

We have,

$$\text{L.H.S.} = \frac{\operatorname{cosec} A - \sin A}{\operatorname{cosec} A + \sin A} = \frac{\frac{1}{\sin A} - \sin A}{\frac{1}{\sin A} + \sin A}$$

$$= \frac{1-\sin^2 A}{1+\sin^2 A} = \frac{\frac{1-\sin^2 A}{\cos^2 A}}{\frac{1+\sin^2 A}{\cos^2 A}} = \frac{\sec^2 A - \tan^2 A}{\sec^2 A + \tan^2 A} = \text{R.H.S.}$$

69.

Given, $\sin\theta = \frac{12}{13} \Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}$

$= \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{\frac{169-144}{169}} = \frac{5}{13}$

$\therefore \quad \cos\theta = \frac{5}{13}$ and $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12}{5}$

Now, $\frac{\sin^2\theta-\cos^2\theta}{2\sin\theta\cos\theta} \times \frac{1}{\tan^2\theta} = \frac{\left(\frac{12}{13}\right)^2-\left(\frac{5}{13}\right)^2}{2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$

$= \frac{\frac{144}{169}-\frac{25}{169}}{\frac{120}{169}} \times \frac{1}{\frac{144}{25}} = \frac{119}{120} \times \frac{25}{144} = \frac{595}{3456}$

70.

We have,

L.H.S. = $\sin^2\theta\tan\theta + \cos^2\theta\cot\theta + 2\sin\theta\cos\theta$

$= \sin^2\theta\frac{\sin\theta}{\cos\theta} + \cos^2\theta\frac{\cos\theta}{\sin\theta} + 2\sin\theta\cos\theta$

$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta} + 2\sin\theta\cos\theta = \frac{\sin^4\theta+\cos^4\theta}{\cos\theta\sin\theta} + 2\sin\theta\cos\theta$

$= \frac{\sin^4\theta+\cos^4\theta+2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{(\sin^2\theta+\cos^2\theta)^2}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$ ...(i)

Now, R.H.S. = $\tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$ ...(ii)

From (i) and (ii), L.H.S. = R.H.S.

71.

We have, L.H.S. = $\frac{1}{\text{cosec}\theta+\cot\theta}-\frac{1}{\sin\theta}$

$= \frac{1}{\text{cosec}\theta+\cot\theta}\times\frac{\text{cosec}\theta-\cot\theta}{\text{cosec}\theta-\cot\theta}-\frac{1}{\sin\theta}$

$= \frac{\text{cosec}\theta-\cot\theta}{\text{cosec}^2\theta-\cot^2\theta}-\frac{1}{\sin\theta}$ $= \text{cosec}\,\theta - \cot\theta - \text{cosec}\,\theta = -\cot\theta$

Now, R.H.S. = $\frac{1}{\sin\theta}-\frac{1}{\text{cosec}\theta-\cot\theta}$

$= \frac{1}{\sin\theta}-\frac{1}{\text{cosec}\theta-\cot\theta}\times\frac{\text{cosec}\theta+\cot\theta}{\text{cosec}\theta+\cot\theta}$

$= \text{cosec}\,\theta - \frac{\text{cosec}\theta+\cot\theta}{\text{cosec}^2\theta-\cot^2\theta}$ $= \text{cosec}\,\theta - \text{cosec}\,\theta - \cot\theta$

$= -\cot\theta$ $[\because \text{cosec}^2\theta - \cot^2\theta = 1]$

$\therefore$ L.H.S. = R.H.S.

72.

We have $1 + \sin^2\theta = 3\sin\theta\cos\theta$

$\sin^2\theta + \cos^2\theta + \sin^2\theta = 3\sin\theta\cos\theta$

$2\sin^2\theta + \cos^2\theta = 3\sin\theta\cos\theta$ ...(i)

On dividing equation (i) each term by $\cos^2\theta$, we get

$2\tan^2\theta + 1 = 3\tan\theta$

$2\tan^2\theta - 2\tan\theta - \tan\theta + 1 = 0$

$2\tan\theta(\tan\theta - 1) - 1(\tan\theta - 1) = 0$

$\Rightarrow$ $(\tan\theta - 1)(2\tan\theta - 1) = 0$

$\Rightarrow$ If $\tan\theta - 1 = 0 \Rightarrow \tan\theta = 1$

$\Rightarrow$ If $2\tan\theta - 1 = 0 \Rightarrow \tan\theta = \frac{1}{2}$

73.

We have,

$$\text{L.H.S.} = \frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A} - 1} + \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A} = \frac{\sin^2 A + \cos^2 A}{\sin^2 A - \cos^2 A}$$

$$= \frac{1}{\sin^2 A - \cos^2 A} \qquad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{1}{1 - \cos^2 A - \cos^2 A} = \frac{1}{1 - 2\cos^2 A} = \text{R.H.S.}$$

74.

We know that $\text{cosec}^2 A = 1 + \cot^2 A$

$\Rightarrow \quad \text{cosec}\, A = \sqrt{1 + \cot^2 A} \quad \Rightarrow \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$

Now, $\sec^2 A = 1 + \tan^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$

$\Rightarrow \quad \sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$ and $\cos A = \frac{\cot A}{\sqrt{\cot^2 A + 1}}$

75.

We have, $\sin A + \sin^3 A = \cos^2 A$

$\Rightarrow \quad \sin A(1 + \sin^2 A) = \cos^2 A$

Squaring both sides, we get $\sin^2 A(1 + \sin^2 A)^2 = \cos^4 A$

$\Rightarrow \quad (1 - \cos^2 A)(1 + (1 - \cos^2 A))^2 = \cos^4 A$

$\Rightarrow \quad (1 - \cos^2 A)(2 - \cos^2 A)^2 = \cos^4 A$

$\Rightarrow \quad (1 - \cos^2 A)(4 + \cos^4 A - 4\cos^2 A) = \cos^4 A$

$\Rightarrow \quad 4 + \cos^4 A - 4\cos^2 A - 4\cos^2 A - \cos^6 A + 4\cos^4 A = \cos^4 A$

$\Rightarrow \quad \cos^6 A - 4\cos^4 A + 8\cos^2 A = 4$

76.

We have,

L.H.S. = $(\cot A + \sec B)^2 - (\tan B - \text{cosec } A)^2$

$$= \left(\frac{\cos A}{\sin A}+\frac{1}{\cos B}\right)^2 - \left(\frac{\sin B}{\cos B}-\frac{1}{\sin A}\right)^2$$

$$= \frac{\cos^2 A}{\sin^2 A}+\frac{1}{\cos^2 B}+\frac{2\cos A}{\sin A\cos B}-\frac{\sin^2 B}{\cos^2 B}-\frac{1}{\sin^2 A}+\frac{2\sin B}{\cos B\sin A}$$

$$= \left(-\frac{1}{\sin^2 A}+\frac{\cos^2 A}{\sin^2 A}\right)+\left(-\frac{\sin^2 B}{\cos^2 B}+\frac{1}{\cos^2 B}\right) +2\left(\frac{\cos A}{\sin A\cos B}+\frac{\sin B}{\cos B\sin A}\right)$$

$$= -\left(\frac{1-\cos^2 A}{\sin^2 A}\right)+\left(\frac{1-\sin^2 B}{\cos^2 B}\right)+2(\cot A\sec B+\tan B\,\text{cosec}\,A)$$

$= -1 + 1 + 2(\cot A \sec B + \tan B \text{ cosec } A)$

$= 2(\cot A \sec B + \tan B \text{ cosec } A)$ = R.H.S.

77.

We have, $x = \sec A - \tan A$

$$\Rightarrow \quad x = \frac{1}{\cos A}-\frac{\sin A}{\cos A} = \frac{1-\sin A}{\cos A} \qquad \text{...(i)}$$

$$\text{Now, L.H.S.} = \frac{x^2+1}{x^2-1} = \frac{\frac{(1-\sin A)^2}{\cos^2 A}+1}{\frac{(1-\sin A)^2}{\cos^2 A}-1} \qquad \text{[Using (i)]}$$

$$= \frac{\frac{1+\sin^2 A-2\sin A+\cos^2 A}{\cos^2 A}}{\frac{1+\sin^2 A-2\sin A-\cos^2 A}{\cos^2 A}} = \frac{1+(\sin^2 A+\cos^2 A)-2\sin A}{(1-\cos^2 A)+\sin^2 A-2\sin A}$$

$$= \frac{1+1-2\sin A}{\sin^2 A+\sin^2 A-2\sin A} = \frac{2(1-\sin A)}{2\sin^2 A-2\sin A}$$

$$= \frac{2(1-\sin A)}{-2\sin A(1-\sin A)} = -\frac{1}{\sin A} = -\text{cosec}\,A = \text{R.H.S.}$$

78.

We have, $\frac{\operatorname{cosec}A-\cot A}{\operatorname{cosec}A+\cot A}+\frac{\operatorname{cosec}A+\cot A}{\operatorname{cosec}A-\cot A}$

$$= \frac{\frac{1}{\sin A}-\frac{\cos A}{\sin A}}{\frac{1}{\sin A}+\frac{\cos A}{\sin A}}+\frac{\frac{1}{\sin A}+\frac{\cos A}{\sin A}}{\frac{1}{\sin A}-\frac{\cos A}{\sin A}} = \frac{\frac{1-\cos A}{\sin A}}{\frac{1+\cos A}{\sin A}}+\frac{\frac{1+\cos A}{\sin A}}{\frac{1-\cos A}{\sin A}}$$

$$= \frac{1-\cos A}{1+\cos A}+\frac{1+\cos A}{1-\cos A} = \frac{(1-\cos A)^2+(1+\cos A)^2}{(1+\cos A)(1-\cos A)}$$

$$= \frac{1+\cos^2 A-2\cos A+1+\cos^2 A+2\cos A}{1-\cos^2 A}$$

$$= \frac{2+2\cos^2 A}{1-\cos^2 A} = \frac{2(1+\cos^2 A)}{1-\cos^2 A} \qquad \text{...(i)}$$

Also, $2(2\operatorname{cosec}^2 A - 1)$

$$= 2\left(\frac{2}{\sin^2 A}-1\right) = 2\left(\frac{2-\sin^2 A}{1-\cos^2 A}\right)$$

$$= 2\left(\frac{2-(1-\cos^2 A)}{1-\cos^2 A}\right) = 2\left(\frac{1+\cos^2 A}{1-\cos^2 A}\right) \qquad \text{...(ii)}$$

From (i) and (ii), we get $\frac{\operatorname{cosec}A-\cot A}{\operatorname{cosec}A+\cot A}+\frac{\operatorname{cosec}A+\cot A}{\operatorname{cosec}A-\cot A}$

$$= 2(2\operatorname{cosec}^2 A - 1) = 2\left(\frac{1+\cos^2 A}{1-\cos^2 A}\right)$$

79.

Given, $m = \cos A - \sin A$, $n = \cos A + \sin A$

Now, $\frac{m}{n} - \frac{n}{m} = \frac{m^2 - n^2}{mn} = \frac{(\cos A - \sin A)^2 - (\cos A + \sin A)^2}{(\cos A - \sin A)(\cos A + \sin A)}$

$= \frac{\cos^2 A + \sin^2 A - 2\cos A \sin A - \cos^2 A \quad - \sin^2 A - 2\sin A \cos A}{\cos^2 A - \sin^2 A}$

$= \frac{-4\sin A \cos A}{\cos^2 A - \sin^2 A}$ ...(i)

Divide numerator and denominator by $\sin A \cos A$, we get,

$\frac{-4}{\frac{\cos^2 A}{\sin A \cos A} - \frac{\sin^2 A}{\sin A \cos A}} = \frac{-4}{\cot A - \tan A}$ ...(ii)

From (i) and (ii), we get $\frac{m}{n} - \frac{n}{m} = \frac{-4\sin A \cos A}{\cos^2 A - \sin^2 A} = \frac{-4}{\cot A - \tan A}$

80.

We have, L.H.S. $= \frac{\sec^3\theta}{\sec^2\theta - 1} + \frac{\operatorname{cosec}^3\theta}{\operatorname{cosec}^2\theta - 1}$

$= \frac{\frac{\sec^3\theta}{\sec^2\theta}}{\frac{\sec^2\theta - 1}{\sec^2\theta}} + \frac{\frac{\operatorname{cosec}^3\theta}{\operatorname{cosec}^2\theta}}{\frac{\operatorname{cosec}^2\theta - 1}{\operatorname{cosec}^2\theta}} = \frac{\sec\theta}{1 - \cos^2\theta} + \frac{\operatorname{cosec}\theta}{1 - \sin^2\theta}$

$= \frac{\sec\theta}{\sin^2\theta} + \frac{\operatorname{cosec}\theta}{\cos^2\theta} = \sec\theta \operatorname{cosec}^2\theta + \operatorname{cosec}\theta \sec^2\theta$

$= \sec\theta \operatorname{cosec}\theta (\sec\theta + \operatorname{cosec}\theta) =$ R.H.S.

81.

We have,

L.H.S. = (tan θ + sec θ – 1)(tan θ + 1 + sec θ)

$$= \left(\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}-1\right)\left(\frac{\sin\theta}{\cos\theta}+1+\frac{1}{\cos\theta}\right)$$

$$= \left(\frac{\sin\theta+1-\cos\theta}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta+1}{\cos\theta}\right)$$

$$= \frac{\sin^2\theta+\sin\theta\cos\theta+\sin\theta+\sin\theta+\cos\theta+1 -\sin\theta\cos\theta-\cos^2\theta-\cos\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta+2\sin\theta+1-\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta+2\sin\theta+\sin^2\theta}{\cos^2\theta}=\frac{2\sin^2\theta+2\sin\theta}{\cos^2\theta}$$

$$= \frac{2\sin\theta(1+\sin\theta)}{1-\sin^2\theta}=\frac{2\sin\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}=\frac{2\sin\theta}{1-\sin\theta} = \text{R.H.S.}$$

82.

We have, L.H.S. = $\sqrt{\sec^2\theta+\text{cosec}^2\theta}$

$$= \sqrt{(1+\tan^2\theta)+(1+\cot^2\theta)} = \sqrt{\tan^2\theta+\cot^2\theta+2}$$

$$= \sqrt{\tan^2\theta+\cot^2\theta+2\tan\theta\cot\theta} \qquad [\because \tan\theta\cot\theta = 1]$$

$$= \sqrt{(\tan\theta+\cot\theta)^2} = (\tan\theta + \cot\theta) = \text{R.H.S.}$$

83.

We have, $\tan\theta + \sin\theta = m$ ...(i)

and $\tan\theta - \sin\theta = n$ ...(ii)

Squaring (i) and (ii) and then subtracting, we get

$m^2 - n^2 = \tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta$

$- \tan^2\theta - \sin^2\theta + 2\tan\theta\sin\theta$

$\Rightarrow \quad m^2 - n^2 = 4\tan\theta\sin\theta$ ...(iii)

Multiplying (i) and (ii), we get $\tan^2\theta - \sin^2\theta = mn$

$$\Rightarrow \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta = mn \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\Rightarrow \frac{\sin^2\theta}{\cos^2\theta}(1-\cos^2\theta) = mn$$

$$\Rightarrow \tan^2\theta\sin^2\theta = mn \Rightarrow \tan\theta\sin\theta = \sqrt{mn}$$

Using above value in (iii), we get $m^2 - n^2 = 4\sqrt{mn}$

CBSE Sample Questions

1.

(a): Given, $\tan\beta = \frac{4}{5}$

So, $$\frac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta} = \frac{5\tan\beta - 2}{5\tan\beta + 2} = \frac{5\times\frac{4}{5} - 2}{5\times\frac{4}{5} + 2} = \frac{1}{3}$$ **(1)**

2.

(a): We have, $\frac{4\sin\beta - 3\cos\beta}{4\sin\beta + 3\cos\beta}$

Dividing both numerator and denominator by $\cos\beta$, we get

$$\frac{4\tan\beta - 3}{4\tan\beta + 3} = \frac{3-3}{3+3} = 0 \quad \left(\because \tan\beta = \frac{3}{4} \text{(given)}\right)$$ **(1)**

3.

(b): We have, $\tan\alpha + \cot\alpha = 2$

$$\Rightarrow \tan\alpha + \frac{1}{\tan\alpha} = 2$$

$$\Rightarrow \tan^2\alpha - 2\tan\alpha + 1 = 0 \Rightarrow (\tan\alpha - 1)^2 = 0$$

$$\Rightarrow \tan\alpha = 1 \qquad \text{...(i)}$$

$$\therefore \tan^{20}\alpha + \cot^{20}\alpha = (\tan\alpha)^{20} + \left(\frac{1}{\tan\alpha}\right)^{20} = 1 + \left(\frac{1}{1}\right)^{20} = 2$$

(1)

4.

(b): $\because \angle C = 90°$,

$$\therefore \frac{\cot y°}{\cot x°} = \frac{AC/BC}{AC/CD} = \frac{CD}{BC} = \frac{CD}{2CD}$$

[$\because$ D is the midpoint of $BC \Rightarrow BD = CD$]

$$= \frac{1}{2}$$

(1)

5.

Given, $\tan A = \frac{3}{4} = \frac{3k}{4k}$ (say)

$\Rightarrow$ $P = 3k, B = 4k$

$\Rightarrow$ $H = \sqrt{9k^2 + 16k^2} = 5k$ **(1/2)**

$\therefore$ $\sin A = \frac{3k}{5k} = \frac{3}{5}, \cos A = \frac{4k}{5k} = \frac{4}{5}$ **(1/2)**

So, $\frac{1}{\sin A} + \frac{1}{\cos A} = \frac{1}{3/5} + \frac{1}{4/5}$ **(1/2)**

$= \frac{5}{3} + \frac{5}{4}$

$= \frac{20+15}{12} = \frac{35}{12}$ **(1/2)**

6.

(b): Given, $x \tan 60° \cos 60° = \sin 60° \cot 60°$

$$\Rightarrow \quad x \times \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{1}{\sqrt{3}} = \tan 30° \qquad \textbf{(1)}$$

7.

(b): We know that, $\tan A = \sqrt{3} \Rightarrow \angle A = 60°$.

Also, $\angle B = 90°$

$\therefore \quad \angle A + \angle B + \angle C = 180° \Rightarrow \angle C = 30°$

So, $\cos A \cos C - \sin A \sin C = \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} = 0$ **(1)**

8.

(a): Let measure of $\angle A$, $\angle B$ and $\angle C$ be x, x and $2x$ respectively.

$\therefore \quad x + x + 2x = 180° \Rightarrow x = 45°$

So, $\angle A$, $\angle B$ and $\angle C$ are 45°, 45° and 90° respectively.

$$\therefore \frac{\sec A}{\operatorname{cosec} B} - \frac{\tan A}{\cot B} = \frac{\sec 45°}{\operatorname{cosec} 45°} - \frac{\tan 45°}{\cot 45°}$$

$$= \frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{1} = 1 - 1 = 0 \qquad \textbf{(1)}$$

9.

Given, $\sin A + \cos B = 1$

$\Rightarrow \quad \sin 30° + \cos B = 1$ $\quad [\because A = 30° \text{ (Given)}]$

$\Rightarrow \quad \frac{1}{2} + \cos B = 1$ **(1/2)**

$\Rightarrow \quad \cos B = \frac{1}{2} = \cos 60° \Rightarrow B = 60°$ **(1/2)**

10.

We have given, $\sin(A+B) = 1 \Rightarrow \sin(A+B) = \sin 90°$,

$\Rightarrow$ $A + B = 90$ ...(i) **(1/2)**

$\cos(A - B) = \dfrac{\sqrt{3}}{2} = \cos 30° \Rightarrow A - B = 30°$ ...(ii) **(1/2)**

From (i) & (ii) $\angle A = 60°$ and $\angle B = 30°$ **(1)**

11.

We have, $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \dfrac{1-\sqrt{3}}{1+\sqrt{3}}$

Dividing the numerator and denominator of LHS by $\cos\theta$, we get

$\dfrac{1-\tan\theta}{1+\tan\theta} = \dfrac{1-\sqrt{3}}{1+\sqrt{3}}$ **(1)**

Which on simplification (or comparison) gives $\tan\theta = \sqrt{3}$

We know that $\tan 60° = \sqrt{3}$ $\therefore$ $\theta = 60°$ **(1)**

12.

Given, $\sqrt{3}\sin\theta = \cos\theta$ **(1/2)**

$\Rightarrow$ $\dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\sqrt{3}}$ **(1/2)**

$\Rightarrow$ $\tan\theta = \dfrac{1}{\sqrt{3}} = \tan 30°$ **(1/2)**

$\Rightarrow$ $\theta = 30°$ **(1/2)**

13.

(b): $\sin\theta + \cos\theta = \sqrt{2}$

Squaring both sides, we get

$(\sin\theta + \cos\theta)^2 = 2$

$\Rightarrow$ $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2 \Rightarrow 1 + 2\sin\theta\cos\theta = 2$

$[\because \sin^2\theta + \cos^2\theta]$

$\Rightarrow$ $\sin\theta \cdot \cos\theta = \dfrac{1}{2}$ ...(i)

Now, $\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$= \dfrac{1}{\sin\theta \cdot \cos\theta} = 2$ [From (i)] **(1)**

14.

(b): We have, $2\sin^2\beta - \cos^2\beta = 2$

$\Rightarrow \quad 2\sin^2\beta - (1 - \sin^2\beta) = 2$

$\Rightarrow \quad 3\sin^2\beta = 3 \Rightarrow \sin^2\beta = 1 \Rightarrow \beta = 90°$ **(1)**

15.

(c) : We have, $1 + \sin^2\alpha = 3\sin\alpha\cos\alpha$

$\Rightarrow \quad \sin^2\alpha + \cos^2\alpha + \sin^2\alpha = 3\sin\alpha\cos\alpha$

$(\because \sin^2\alpha + \cos^2\alpha = 1)$

$\Rightarrow \quad 2\sin^2\alpha - 3\sin\alpha\cos\alpha + \cos^2\alpha = 0$

$\Rightarrow \quad 2\sin^2\alpha - 2\sin\alpha\cos\alpha - \sin\alpha\cos\alpha + \cos^2\alpha = 0$

$\Rightarrow \quad 2\sin\alpha = \cos\alpha$ or $\sin\alpha = \cos\alpha$

$\Rightarrow \quad \cot\alpha = 2$ or $\cot\alpha = 1$ **(1)**

16.

Consider, $x + y = 2\sin^2\theta + 2\cos^2\theta + 1$ **(1/2)**

$[\because x = 2\sin^2\theta, y = 2\cos^2\theta + 1$ (Given)$]$

$= 2(\sin^2\theta + \cos^2\theta) + 1$

$= 2 + 1 = 3$ $[\because \sin^2\theta + \cos^2\theta = 1]$ **(1/2)**

17.

$$\text{L.H.S.} = \frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}$$

$$= \frac{\sin^3\theta/\cos^3\theta}{1+\sin^2\theta/\cos^2\theta}+\frac{\cos^3\theta/\sin^3\theta}{1+\cos^2\theta/\sin^2\theta}$$

$$\left(\because\ \tan\theta=\frac{\sin\theta}{\cos\theta},\cot\theta=\frac{\cos\theta}{\sin\theta}\right)$$ **(1/2)**

$$= \frac{\sin^3\theta/\cos^3\theta}{(\cos^2\theta+\sin^2\theta)/\cos^2\theta}+\frac{\cos^3\theta/\sin^3\theta}{(\sin^2\theta+\cos^2\theta)/\sin^2\theta}$$ **(1/2)**

$$=\frac{\sin^3\theta}{\cos\theta}+\frac{\cos^3\theta}{\sin\theta} = \frac{\sin^4\theta+\cos^4\theta}{\cos\theta\sin\theta}$$ **(1/2)**

$$=\frac{(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$$ **(1/2)**

$$=\frac{1-2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$$=\frac{1}{\cos\theta\sin\theta}-\frac{2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$$

$= \sec\theta \operatorname{cosec}\theta - 2\sin\theta \cos\theta =$ R.H.S. **(1)**